B.Sc (Math) — part - I
paper — II
Topic: - The straight line (In space)
Solved problem

① Find the distance of the point $(-1,-5,-10)$ from the point of the intersection of line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$

Soln: — The eqn. of lines are
$$\frac{x-2}{3}=\frac{y+1}{3}=\frac{z-2}{12}=K \text{ (say)}$$
$x-2=3K,\ y+1=4K,\ z-2=12K$
$\therefore$ Any point on the line is $(3K+2,\ 4K-1,\ 12K+2)$
If this point lies on the plane $x-y+z=5$
we get $3K+2-(4K-1)+(12K+2)=5$
$\Rightarrow 3K+2-4K+1+12K+2=5$
$\Rightarrow 11K+5=5 \Rightarrow 11K=0 \quad \therefore K=0$
The point of intersection of line with the plane is $(2,-1,2)$
$\therefore$ The required distance
$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}$
$=\sqrt{9+16+144}=\sqrt{169}=13$

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problem (2) Find the equation of the line through (1,2,3) and parallel to the line $x-y+2z=5$, $3x+y+z=6$

Soln :— The equation of given lines are

$x-y+2z=5$ —— (1)

and $3x+y+z=6$ —— (2)

Let $l, m, n$ be the direction cosines of the line which passes through the point (1,2,3)

Therefore, eqn. of line is

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{n}.$$

This line is parallel to the line of intersection of the given planes, the normal to the planes will be perpendicular to the line.

we have $l-m+2n=0$

and $3l+m+n=0$

$$\frac{l}{-1-2} = \frac{m}{6-1} = \frac{n}{1+3}$$

or $$\frac{l}{-3} = \frac{m}{5} = \frac{n}{4}$$

Hence the required eqn. of

Line is

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

Problem ③ prove that the straight lines

$$\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4} \text{ and}$$

$$x+2y-3z-14 = 0 = 3x+4y+5z-26$$

Soln. The equation of given lines is

$\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$ —① and plane are

$x+2y-3z=14$ —② and $3x+4y+5z-26=0$ —③

Let $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4} = r$, $\therefore x = 2r-1$
$y = 3r-1$
$z = 4r-1$

Any point on the line ① is $(2r-1, 3r-1, 4r-1)$

If this point lies on plane ② then

$2r-1+6r-2+12r-3=14$

& $20r=20$, $r=1$

The point of intersection ① and ② is $(2-1, 3-1, 4-1)$ i.e. $(1,2,3)$

Substituting this point on the line ③ we get

$3 \cdot 1 + 4 \cdot 2 + 5 \cdot 3 = 26$ or $26 = 26$

which is satisfied

Hence the given lines are coplanar and their point of intersection $(1,2,3)$.